University of Toronto

Department of Electrical and Computer Engineering

ECE 342 - Digital Hardware

Midterm Examination

February 2000

Last Name: _____

First Name: _____

Student Number: _____

Signature: _____

Duration: **1 Hour and 20 minutes**

Answer ALL questions on this test paper. There is extra space at the end if you need it.

EXAMINER's REPORT

1	_____	/8
2	_____	/10
3	_____	/12
4	_____	/9
5	_____	/8
6	_____	/13
TOTAL:	_____	/60

Question 1 — [8 marks]

a. Evaluate the following expressions:

i. 0x0 * 111 = _____

ii 11x * 10x = _____

iii. 0x0 * x11 = _____

iv. 01xx * xx01 = _____

v. x11 # 0xx = _____

vi. 0xx # x11 = _____

vii. 1xx1x0 # 11x01x = _____

viii. x100x1 # 1x0x0x = _____

b. Consider the cover $C^k = p^1, p^2, p^3, p^4$

i. A cube in the cover is *irredundant* if it is not contained inside any other cube. List below the # operations needed to indentify all redundant cubes in C^k:

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Question 2 — [10 marks]

a. Assume we are given two *k*-cubes, A and B. If A and B have no vertices in common is the following possible?

$C = A * B \neq \phi$? and *C* is also a k-cube?

Circle: YES, NO

If you circled yes, give a suitable example of A and B:

A = _____

B = _____

b. The coordinate #-operation is evaluated using a table whose entries are 0, 1, ε, or ϕ. Assume we have two k-cubes A, B and we evaluate the coordinate #-operation for each pair of digits. If the result is ϕ in exactly one digit then the number of vertices in A that are also in B is:

Circle: k

2^{k-1}

0

1

c. Assume a function is specified with an ON set and DC set. Then a cover is

$C = \{ON\} \cup \{DC\}$

Assume we find the prime implicants of this function to be $\{p^1, \ldots, p^i, \ldots p^n\}$. In class we used the following expression to determine if a cube p^i is essential:

$p^i \# (C - p^i) \# \{DC\} \neq \phi$?

It is postulated that the following is an equivalent test:

$p^i \# (C - p^i) \# \{DC - p^i\} \neq \phi$?

Is this equivalent?

Circle: Yes, No

Explain: _____

_____

Question 3 — [12 marks]

For a function, *f*, consider the following cover, which comprises the prime implicants of *f*.

$f(x_1, x_2, x_3, x_4, x_5) = \{x0xx0, 00xxx, 0xx1x, x00xx, x1x1x, xx01x\}$

Below, list the essential prime implicants of *f*. You must derive your answers in the space on the next page and you must use cubical notation and the # operator.

Essential PIs: _____

_____

_____

_____

_____

_____

List the vertices for which *f* is 1 that are covered by only one prime implicant: _____

_____

_____

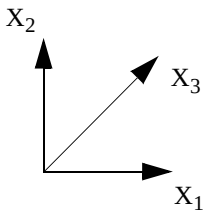
Give a minimal cover for *f* in cubical notation:

f = _____

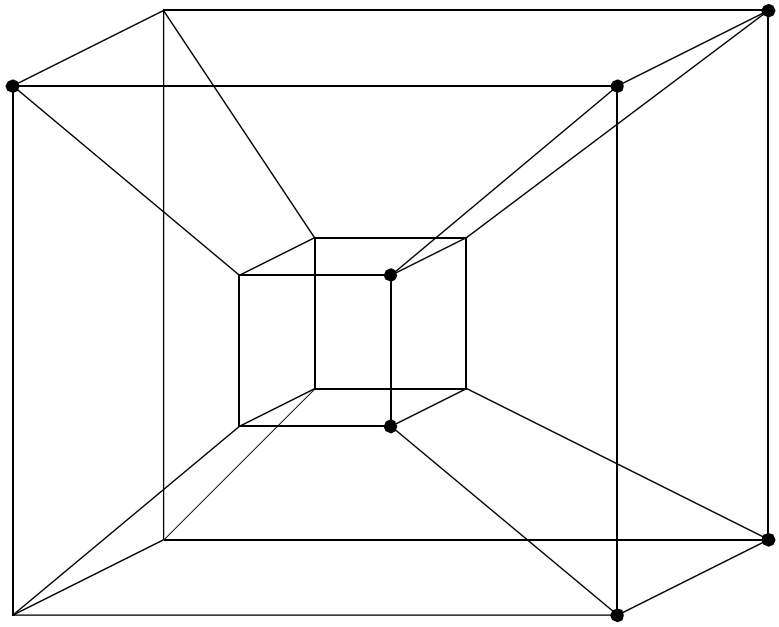
ANSWER SPACE FOR DERIVING THE ESSENTIAL PRIME IMPLICANTS

Question 4 — [9 marks]

The figure below represents the Boolean space (x_1, x_2, x_3, x_4). Coordinates are defined as in class, as indicated below:



(x_4 is 0 on the outer cube)



A function, *f*, is represented by the vertices shown in bold.

a. Write an expression in cubical form that represents a minimal sum-of-products implementation:

f = _____

b. Write an expression in cubical form that corresponds directly to an implementation using only three-input AND operations and an OR operation (of any size). Use as few AND operations as possible:

f = _____

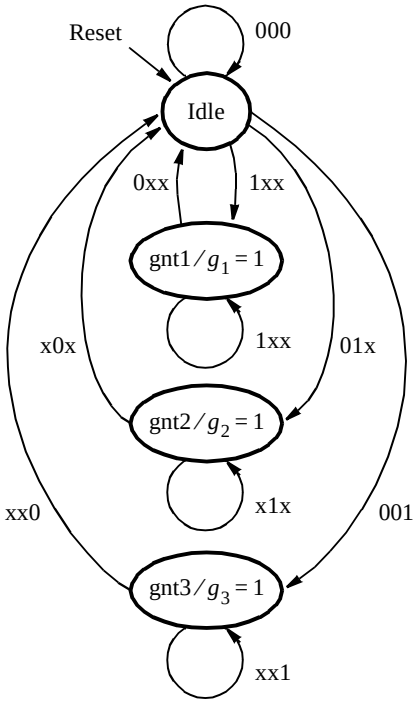
c. Consider a 6-variable Boolean space. One vertex in this space is 101010. If you could draw a figure representing the 6-variable space, which other vertices would be connected by edges to the vertex 101010?

Answer: _____

_____

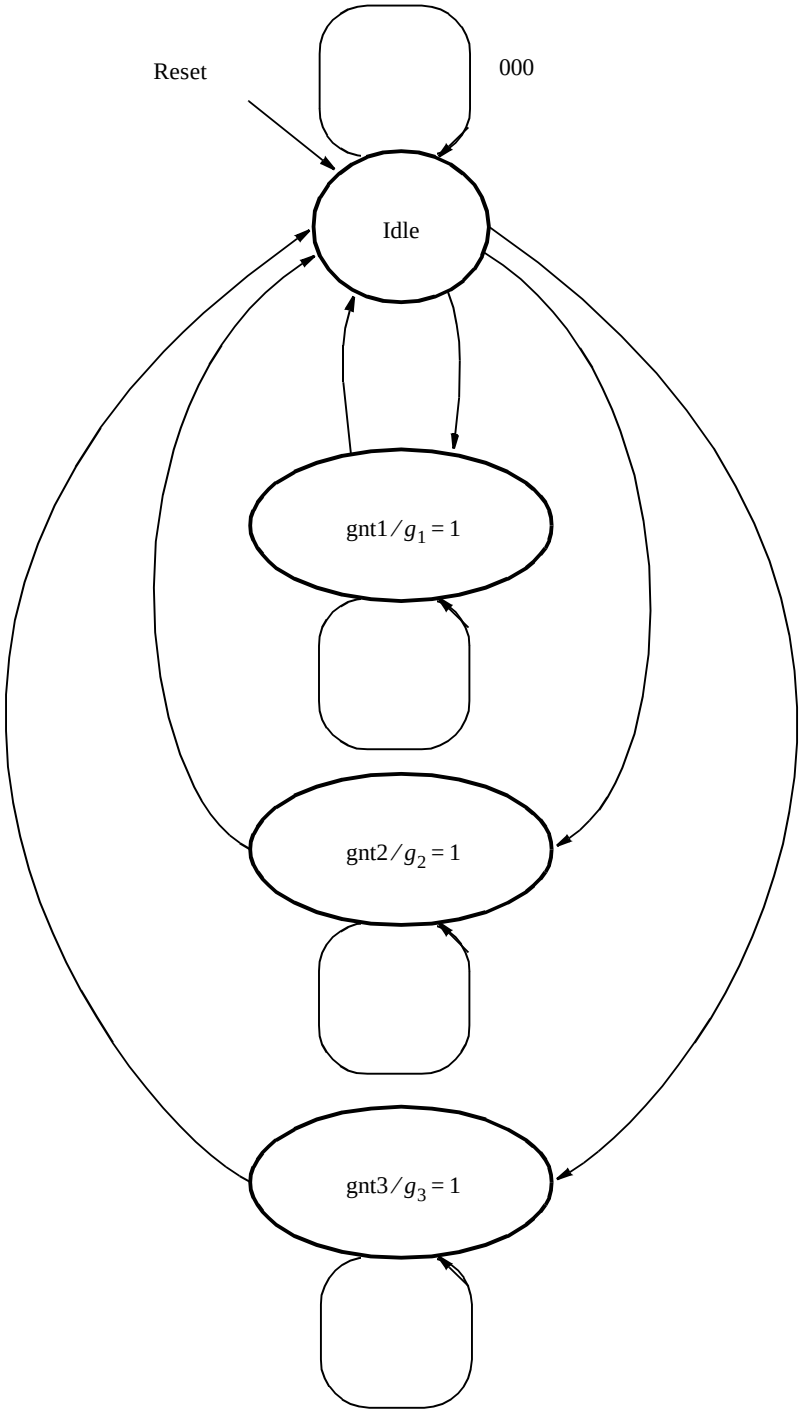
Question 5 — [8 marks]

In class you were shown the state diagram for an arbiter:



a. The arbiter returns to the *Idle* state in between grants. Using this FSM explain under what conditions is it possible that device 3 may not receive a grant for an extended period of time even though it continuously keeps its request active:

b. Label the arcs and draw additional arcs on the state diagram below that solve the above issue, but still maintain the priorty scheme such that if multiple devices make concurrent requests, then device 1 has highest priority, ..., device 3 has lowest.:



Question 6 — [13 marks]

You are to design a circuit that produces:

$$B = \lfloor \log_2 A \rfloor$$

where A is a positive integer and the $\lfloor\ \rfloor$ floor operator rounds down to the nearest integer.

a. Write pseudocode for the operation, using the format given in class. Assume A can be shifted to the right:

b. Draw an ASM chart that represents your pseudocode. Assume an externally-supplied input *s* is provided and is set to 1 when *A* has been loaded and the operation should begin. Set an output *Done* to 1 when the operation is complete.

c. Draw a datapath for your circuit. Label all nodes with reasonable names:

d. For the control circuit, what would be the input and output signals?

Inputs: _____

Outputs: _____

EXTRA SPACE — USE ONLY IF NEEDED

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