

Question 1 – [15 marks]

a. Assume we define the 2-cube $A = x1x1$, and also the 2-cubes $B = xx11$, $C = x0x1$, $D = xx00$, and $E = x0x0$. Consider the $*$ -operations $A * B$, $A * C$, $A * D$, and $A * E$. Which of these operations produces as a result:

- i. A smaller cube: $A \neq B$
- ii. A larger cube: $A \neq C$
- iii. A 2 cube: $A \neq D$
- iv. No cube (\emptyset): $A \neq E$

b. For the 2-cube $A = x1x1$, part a. above considered the $*$ -operation with the 2-cubes B , C , D , and E . For this part, consider all possible 2-cubes. The $*$ -operation of A with each of these possible 2-cubes would produce a resultant cube of a certain size. In the spaces below you are asked to give the total number of different 2-cubes for which the $*$ -operation with A produces a cube of the specified size. Let the other cube in the $*$ -operation be called Z :

- i. $A * Z$ is (\emptyset): # cubes = 1
- ii. $A * Z$ is a 0-cube: # cubes = 4
- iii. $A * Z$ is a 1-cube: # cubes = 4
- iv. $A * Z$ is a 2-cube: # cubes = 4
- v. $A * Z$ is a 3-cube: # cubes = 2

c. For a k -cube, answer the following:

- i. How many 0-cubes are contained in a k -cube: 2^k
- ii. How many $(k-1)$ -cubes are contained in a k -cube: $2k$

d. Given

$$C^{k+1} = \{0x1, x01, x11, 1x1, 11x\} \text{ and}$$

$$G^{k+2} = \{001, 011, x1x, x11, 101, 1x1, 111\}$$

Evaluate the following:

$$C^{k+2} = \{C^{k+1} \cup G^{k+2} - (\text{redundant cubes})\}$$

$$C^{k+2} = \underline{\{x1x, 0x1, x01, 1x1\}}$$

Question 2 – [10 marks]

For a function, f , consider the following cover:

$$f(x_1, x_2, x_3, x_4, x_5) = \{0x11x0, 0x111x, 01x011, 0x110x, 101111\}$$

In the space below, you are to list the prime implicants of f . You must derive your answers in the space on the following page using the procedure discussed in class; specifically, you must use the procedure in which the original cover (given above) is called C^0 and you generate a new cover $C^1 = G^1 \cup C^0$ – (redundant cubes), and so on until the prime implicants are generated.

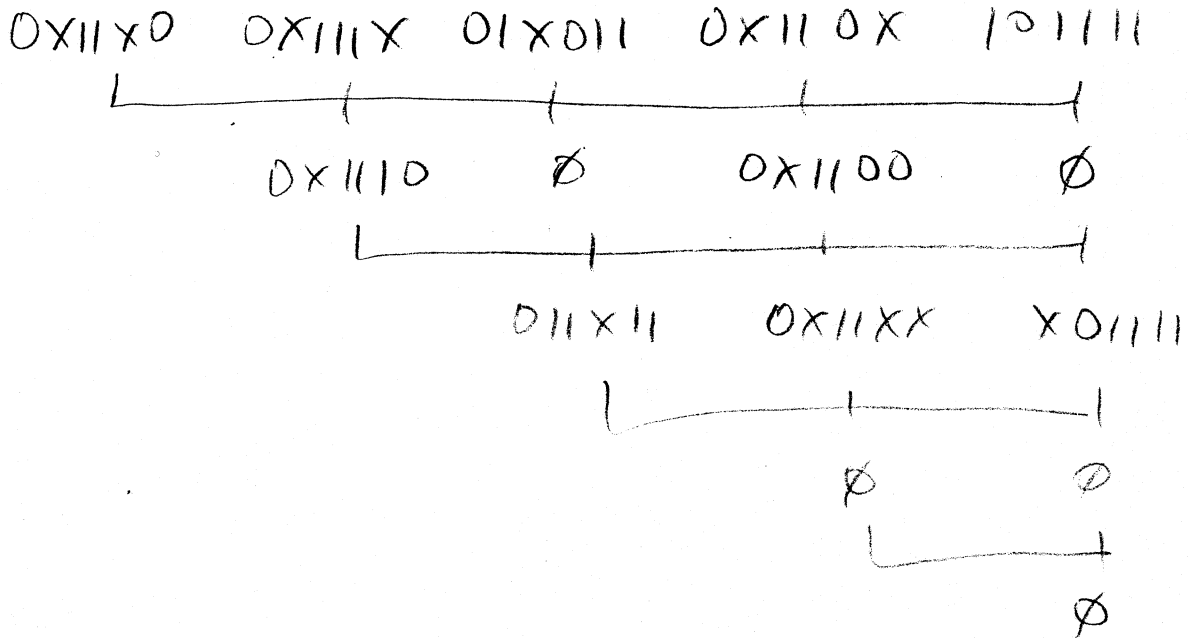
List the prime implicants below and derive them in the space given on the following page.

Prime implicants: $\{0x11xx, 011x11, x01111, 01x011\}$ 7

List the essential prime implicants (you do not have to show your derivation for this part):

$\{0x11xx, 01x011, x01111\}$ 3

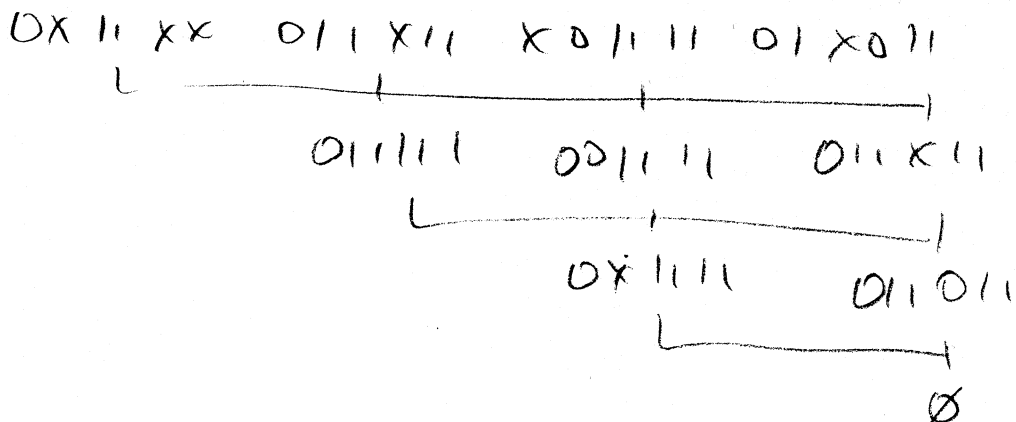
Derive the prime implicants in the space below:



$$G^1 = \{0X1110, 0X1100, 011x11, 0X11XX, x01111\}$$

$$C^2 = C^1 \cup G^1 - (\text{red cubes})$$

$$C^2 = \{0X11XX, 011x11, x01111, 01x011\}$$



$$G^2 = \{011111, 001111, 011x11, 0X1111, 011011\}$$

$$C^3 = C^2 \cup G^2$$

$$C^3 = C^2 = \text{Prime Implicants}$$