

# Question 1 – [15 marks]

a. In a 4-variable Boolean space consider the 2-cube  $A = x1x1$ . There is exactly one other cube, B, for which  $A * B = \emptyset$ :

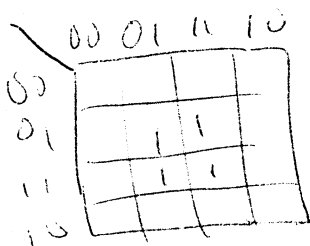
2 i. Give the cube B:      $x0x0$     

2 ii. Is the following statement true or false: For every possible 2-cube, G, there exists exactly one other 2-cube, H, for which  $G * H = \emptyset$ .

ANSWER (circle one):     true     or false

3 iii. Prove or provide a counter example of the following statement: In an n-variable Boolean space, for each (n-1)-cube, K, there exists no other cube, L, for which  $K * L = \emptyset$ .

ANSWER: - for an n-variable cube  $K_{n-1} K_{n-2} \dots K_1 K_0$   
 we assume here that all but one digit  $i = k$ .  
 In order to have  $K * L = \emptyset$ , more than one pair of digits  $K_i * L_i$  have to yield  $\emptyset$ , but this is impossible for the n-1 digits in K that are x. Hence the statement is true



$A = x1x1$

iv. Is the following statement true or false: In an  $n$ -variable Boolean space the number of  $(n-1)$ -cubes that exist is equal to  $n$ .

2 ANSWER (circle one): True or false

v. In a 4-variable Boolean space consider the cube  $A = x_1x_2$ . Define another cube,  $Z$ , such that  $Z \# A = Z$ . How many cubes in total exist that satisfy this constraint?

2 ANSWER for  $Z$ : 00xx (6 others)

2 ANSWER for total number of cubes: 7

vi. Consider a 4-variable Boolean space. For any ~~cube~~<sup>2-cube</sup>  $G$ , how many other cubes,  $H$ , exist such that  $G \# H$  generates multiple cubes?

2 ANSWER: 4

x x x x  
x x x 0

0110  
0110  
0110

### Question 2 – [10 marks]

3 i. Given the following prime implicants for a function, circle the essential prime implicants:

$$f(x_1, x_2, x_3, x_4, x_5, x_6) = \{0x11x0, 0x111x, 01x011, 0x110x, 101111\}$$

4 ii. For a function,  $f$ , consider the following cover:

$$f(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = \{X101XX1, X1011X0, X1010X0, 1X0111X\}$$

List the prime implicants of  $f$ . You can use the space on the next page for rough work if needed.

Prime implicants: X101XX, 1X0111X

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3 Give a minimal cover for  $f$ :

$f = \{PI_5\}$

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