ECE311S

Dynamic Systems and Control

Midterm Exam Solution

Winter 2004

Problem 1

(a) The poles of the system are the roots of

$$s^4 + Ks^3 + (Kb + K)s^2 + K(a+b)s + Kab.$$

Form the Routh array:

$$\begin{array}{c|ccc} s^4 & 1 & Kb+K & Kab \\ s^3 & K & K(a+b) & 0 \\ s^2 & Kb+K-(a+b) & Kab & 0 \\ s & \dfrac{[Kb+K-(a+b)]K(a+b)-K^2ab}{Kb+K-(a+b)} & 0 & 0 \\ s^0 & Kab & 0 & 0 \end{array}$$

Conditions for BIBO stability are obtained by imposing that all entries in the first column of the Routh array be positive, i.e., (recall that we already have that $K > 0, a > 0, b > 0$),

$$Kb + K - (a+b) > 0$$
$$[Kb + K - (a+b)]K(a+b) - K^2ab > 0.$$

(b) Just apply the Final Value Theorem to obtain the following:

$$y_{ss} = \lim_{s\to 0} sY(s) = 1.$$

Problem 2

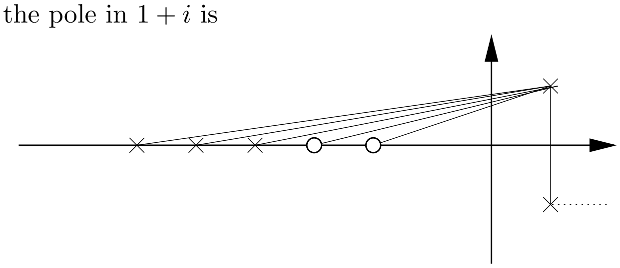
We look at

$$G(s) = \frac{(s+2)(s+3)}{(s+4)(s+5)(s+6)(s^2-2s+2)}$$

and realize that we have 5 branches, two of which approach the zeros in -2 and -3, so there are $n-m = 5-2 = 3$ asymptotes with angles $\pi/3$, π, and $5/3\pi$. The center of the asymptotes is at

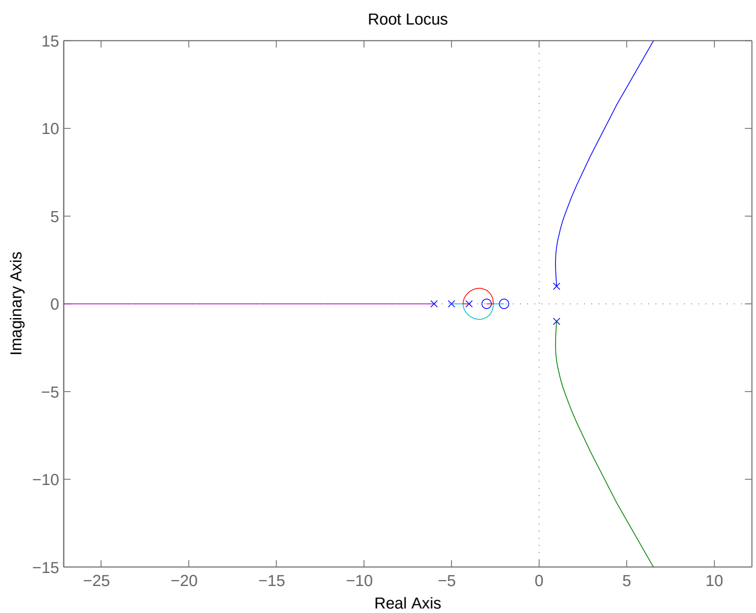
$$\alpha = \frac{(-4-5-6+1+1)-(-2-3)}{3} = -\frac{8}{3}.$$

The departure angle from the pole in $1+i$ is



$$\begin{aligned}\phi_{dep} &= \psi_1 + \psi_2 - \phi_1 - \phi_2 - \phi_3 - \phi_4 \\ &= \arctan(1/3) + \arctan(1/4) - \arctan(1/5) - \arctan(1/6) - \arctan(1/7) - \pi/2 - \pi \\ &= 93.57^\circ.\end{aligned}$$

The root locus is

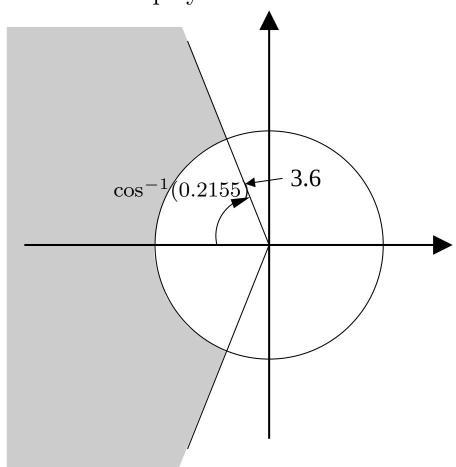


Problem 3

(a) In order to meet the performance specifications, the TWO poles of the closed-loop system must have the following properties:

$$w_n > 3.6 \text{ and } \zeta > 0.2155.$$

Geometrically, the poles of the closed-loop system should lie in the shaded region below.

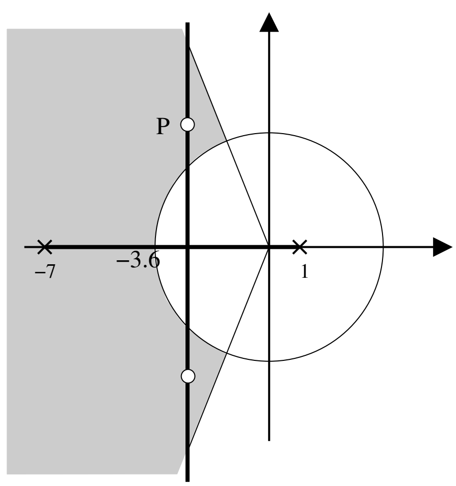


(b) The poles of the closed-loop system are the roots of

$$s^2 + 6s + K - 7,$$

$$s_{1,2} = -3 \pm \sqrt{16 - K}. \tag{1}$$

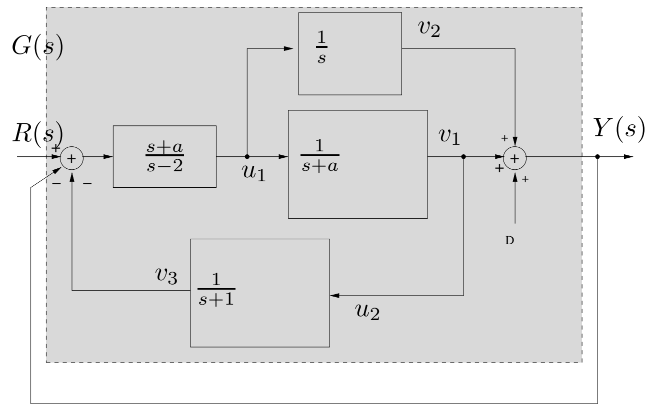
One way to solve the problem is to draw the root locus



and choose a location for the two closed-loop system poles which is compatible with the region found in part (a) of the problem. For instance, we can choose closed-loop system poles at $P = 3 + 4j$ (and its complex conjugate) as in the figure above. So, referring to (1), the problem is solved by setting $\sqrt{16 - K} = 4j$, or $K = 32$.

Problem 4

Trivial state space to transfer function conversions yield the following block diagram:

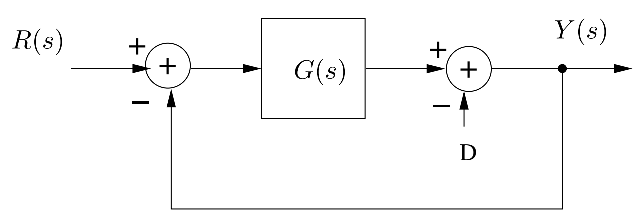


(a) Setting $D(s) = 0$, the transfer function of the system inside the shaded area is

$$G(s) = \frac{s+1}{s^2 - s - 2}\frac{2s+a}{s},$$

so the system type is 1.

(b) The block diagram of the system after block simplifications is:



with G defined above. Applying superposition and the Final Value Theorem one easily arrives at the following:

$$e(\infty) = 0.$$