# ECE310 - December 2002 Final Exam

**PROBLEM 1** (12.5 points)

(a) Two periodic signals have periods $T_1$ and $T_2$. What is the period of the sum of the signals? What is the period of the product of the signals?

(b) Two Fourier series have fundamental frequencies $f_1$ and $f_2 = 2f_1$. What is the fundamendal frequency of the sum of these series?

(c) Two periodic signals have periods $T_1$ and $T_2 = \sqrt{2}T_1$. Is the sum of these signals a periodic signal? If so, find the period.

(d) Explain why an impulse $x(t) = A\delta(t)$ cannot be discritized.

(e) Is it possible for an LTI system to give the same output for two different inputs? If so, give an example. If not, prove it.

**PROBLEM 2** (12.5 points)

(a) A lowpass signal $x(t)$ has a maximum frequency of $\omega_m$ i.e. $X(j\omega) = 0$ for $|\omega| > \omega_m$. It is multiplied by $p(t)$ which is a periodic $\pm 1$ square wave with period $T$. From the product $x(t)p(t)$ it is desired to retrieve the signal $x(t)$ using only low pass and band pass filters and multiplication operations by $cos(\omega_0 t)$. Suggest a system that will accomplish this showing clearly the cut off frequencies of the filters and the frequencies of any multipliers used in terms of $T$ and $\omega_m$.

(b) Repeat part (a) if positive pulses of height $+1$ width $\Delta < \frac{T}{2}$ and period $T$ are used.

**PROBLEM 3** (12.5 points)

(a) An LTI system gives an output of $y(t)$ for an input of $x(t)$. Prove that if we put $dx(t)/dt$ into this system, the output must be $dy(t)/dt$.

(b) An input to a system, $x(t)$ is bandlimited ($X(j\omega) = 0$ for $|\omega| > 1$). The corresponding output is

$$y(t) = x(t)cos^2(t) * \frac{sint}{\pi t}$$

Find the impulse response of an LTI system that gives this output $y(t)$ when the input is $x(t)$.

(c) An LTI system has a frequency response function of

$$H(j\omega) = \frac{1}{(1 + j\omega)^2}$$

If the input to the system is $x(t) = te^{-t}u(t)$, what is its output?

**PROBLEM 4** (12.5 points)

A causal LTI system is described by the difference equation

$$y[n] - ay[n-1] = bx[n] + x[n-1]$$

where $a$ is real and less than 1 in magnitude.

(a) Find a value of $b$ such that the frequency response of the system satisfies

$$|H(e^{j\Omega})| = 1, \qquad |\Omega| \leq \pi$$

This kind of system is called an all-pass system, as it does not attenuate the input for any value of $\Omega$.

(b) Find and plot the output of the above all pass system with $a = -\frac{1}{2}$ when the input is

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

¿From this example , we see that a nonlinear change in phase can have a significantly different effect on a signal than the time shift that results from a linear phase.