ECE310 - December 1998 Final Exam

1. Let $x(t)$ denote the input to a system and let $y(t)$ denote the output. For each of the following systems, determine whether the corresponding system is linear, time-invariant, or both. Justify your answer in each case.

2 marks (a) $y(t) = t^2 x(t-1)$

2 marks (b) $y[n] = x[n]x[n-1]$

2 marks (c) $y[n] = x[n+1] - x[n-1]$

2 marks (d) $y(t) = \mathcal{O}d\{x(t)\}$

2 marks (e) $y(t) = 0$

2. Let $x(t)$ be an arbitrary signal (complex-valued in general), with even and odd parts denoted by

$$x_e(t) = \mathcal{E}v\{x(t)\}$$

and

$$x_o(t) = \mathcal{O}d\{x(t)\}.$$

5 marks (a) Show that

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x_e(t)|^2 dt + \int_{-\infty}^{\infty} |x_o(t)|^2 dt$$

5 marks (b) Show that $x_e(t)$ and $x_o(t)$ are orthogonal, i.e.,

$$\int_{-\infty}^{\infty} x_e(t) x_o^*(t) dt = 0.$$

5 marks 3. (a) Show that an ideal differentiator is a linear time-invariant system.

5 marks (b) If the input $x(t)$ to a linear time-invariant system produces the output $y(t)$, what output does the input $\frac{d}{dt}x(t)$ produce? [*Hint:* consider the cascade of an ideal differentiator and the system in question.]

4. Let $x(t)$ be a signal with Fourier transform $X(j\omega) = 0$ for $\omega \geq w_M$. For each of the following signals, determine the minimum sampling rate ω_s that allows perfect reconstruction of the signal from samples taken at integer multiples of $T_s = 2\pi/\omega_s$.

2 marks (a) $x(t)$

2 marks (b) $x(t) + x(t-1)$

2 marks (c) $\frac{d}{dt}x(t)$

2 marks (d) $x^2(t)$

2 marks (e) $x(t)\cos(\omega_M t)$

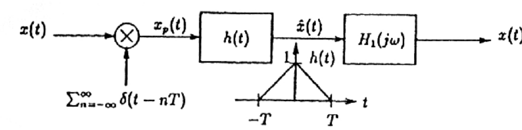

Figure 1: A system for sampling and reconstruction.

5. The system shown in Figure 1 is used for sampling and reconstructing an input signal $x(t)$.

2 marks (a) Let $h(t)$ be an arbitrary impulse response (not necessarily the triangular response shown in the figure). Write an expression in terms of $x(t)$ and $h(t)$ for the signal $\hat{x}(t)$.

4 marks (b) For the remainder of this question, assume $h(t)$ has the specific triangular impulse response shown in Figure 1. Consider the time interval $nT \leq t \leq (n+1)T$, where n is an integer. Show, by writing an explicit equation for $\hat{x}(t)$, that, in the given interval, the only samples of $x(t)$ on which $\hat{x}(t)$ depends are $x(nT)$ and $x((n+1)T)$.

(c) Sketch both $x(t)$ and $\hat{x}(t)$ (on the same graph) for the following signals.

2 marks i. $x(t) = \cos[\pi t/(4T)]$

2 marks ii. $x(t) = \cos[2\pi t/T]$

2 marks

(d) Give conditions on $X(j\omega)$ that allow $x(t)$ to be reconstructed perfectly from $x_p(t)$.

3 marks

(e) Assuming that the conditions in the previous part are satisfied, give an expression for $H_1(j\omega)$ in terms of $H(j\omega)$ so that the overall output of the system is indeed $x(t)$.

5 marks

(f) Find and sketch $H(j\omega)$, and sketch $H_1(j\omega)$.

6. A linear half-wave rectifier is a memoryless system with transfer characteristic

$$R[x(t)] = \begin{cases} x(t), & x(t) > 0, \\ 0, & \text{otherwise.} \end{cases}$$

5 marks

(a) Let $x(t) = \cos(\omega_c t)$. Find a Fourier series representation for $y(t) = R[x(t)]$.

2 marks

(b) Sketch the Fourier transform of $y(t)$.

3 marks

(c) The half-wave rectifier is used to process the standard AM signal $s(t)\cos(\omega_c t)$, where $s(t)$ is a non-negative modulating signal. Show that $R[s(t)\cos(\omega_c t)] = s(t)R[cos(\omega_c t)]$.

5 marks

(d) Assuming that $s(t)$ has bandwidth B, and that $\omega_c > 2B$, show, by sketching the magnitude of the spectrum of $R[s(t)\cos(\omega_c t)]$, that the output of the rectifier has a signal component proportional to $s(t)$.

5 marks

(e) Design a system that will recover $s(t)$ from the rectifier output.