Name: _____ **Student ID:**_____

QUIZ 3, ECE302, Fall 2002, Prof. B. J. Frey
10 marks total; 1 hour; Calculators, notes and textbook not allowed

1. (3 marks total) For the pdf, $f_X(x) = \frac{\alpha}{2}e^{-\alpha|x|}$, $-\infty \leq x \leq \infty$, $\alpha \geq 0$, find the characteristic function $\Phi_X(\omega)$ and find the first and second moments of X using the moment theorem. Recall that

$$\Phi_X(\omega) = \int_{x=-\infty}^{\infty} f_X(x)e^{j\omega x}dx, \quad \mathrm{E}[X^n] = \frac{1}{j^n}\frac{d^n\Phi_X(\omega)}{d\omega^n}\Big|_{\omega=0}.$$

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2. (4 marks total) Suppose the joint pdf for X and Y is $f_{X,Y}(x,y) = c(x^2y^2 - 2xy^2 + x^2 - 2x + y^2 + 1)$, $0 \leq x \leq 1$, $0 \leq y \leq 1$. Determine the constant c, the marginal pdf $f_X(x)$ and the marginal pdf $f_Y(y)$. Are X and Y independent? Justify your answer.

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3. (3 marks total) The number of particles that are absorbed by a sensor on a spacecraft during a time interval t is a Poisson RV with parameter βt. The time required for a particle to be absorbed is an exponential RV with parameter α. Find the pmf for the number of particles N that arrive during the time interval when an an average particle is being absorbed. Show all work.
Formulas. Poisson RV: $P(X = k) = \frac{\gamma^k}{k!}e^{-\gamma}$, $k = 0, 1, \ldots$, $\mathrm{E}[X] = \gamma$, $\mathrm{VAR}[X] = \gamma$. Exponential RV: $f_X(x) = \frac{1}{\gamma}e^{-x/\gamma}$, $x \geq 0$, $\mathrm{E}[X] = \gamma$, $\mathrm{VAR}[X] = \gamma^2$.