

ECE302F Lecture Groups 101 and 102 – Quiz 3

Rules:

- One letter-size aid sheet allowed;
 - Non-programmable electronic calculators CAN be used;
 - Total duration of quiz: 60 minutes;
 - Answer all questions.
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1. Let X_1 and X_2 be independent random variables with pdf

$$f_{X_i}(x_i) = \begin{cases} e^{-x_i}, & x_i \geq 0 \\ 0, & x_i < 0 \end{cases}, \quad i = 1, 2$$

Find the pdf of $Z = X_1 - X_2$. (5 marks)

2. A communication channel randomly transmits a sequence of 3 bits (e.g. $\{1, 0, 1\}$). Each bit takes on the value 0 or 1 with equal probability (50/50). Consider the random variables X and Y . X is the number of 1's found in the 3 bit sequence and Y is defined as

$$Y = \begin{cases} 1, & X \text{ is even} \\ 0, & X \text{ is odd} \end{cases}.$$

Find the covariance of X and Y , $\text{COV}(X, Y)$. (Note that X is a binomial random variable with $n = 3$ and $p = 0.5$.) (5 marks)

3. Let $Y = X + N$, where X is uniformly distributed between 0 and 1, and N is a zero-mean, unit-variance Gaussian random variable. X and N are independent.

- (a) Find $f_Y(y|x)$ and the conditional mean of Y given $X = x$, i.e. $E[Y|x]$. (2 marks)
- (b) Find the mean value of Y , $E[Y]$. (2 marks)
- (c) Find the joint pdf of X and Y , $f_{X,Y}(x, y)$. (2 marks)
- (d) Given that the marginal pdf of Y evaluated at $y = 0.6$ is $f_Y(0.6) = 0.38$, find $f_X(x|y)$ when $y = 0.6$. (2 marks)
- (e) Sketch $f_X(x|y)|_{y=0.6}$ and $f_X(x)$ on the same figure, and use your curves to explain whether knowledge that $Y = 0.6$ improves our knowledge of the value of X . (2 marks)