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QUIZ 2, ECE302, Fall 2002, 5 Marks Total, Prof. B. J. Frey
10 marks total; 45 minutes; Calculators, notes and textbook not allowed

1. (3 marks total) At an engineering party, there are two dimly lit closets labeled "girls" and "boys". In the "girls" closet, there are 2 girls. In the "boys" closet, there are 2 boys and 1 girl. The organizer flips a fair coin to choose which closet door to open. A student enters the closet, chooses a person at random with equal probability, and leads the person *out of the closet* (*i.e.*, "random dating without replacement"). The next student enters the closet with the label corresponding to the sex of the last person chosen, and selects a person at random, and so on. Let B_i and G_i be the events that the "boys" or the "girls" closet is chosen before the ith trial. (So, $P(B_1) = P(G_1) = 0.5$.)

a) (1 mark) Draw the tree diagram for this random experiment and use the tree to compute $P(B_3)$ and $P(G_3)$ (you may leave your answers as fractions).

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b) (1 mark) Use Bayes rule and the tree diagram to compute $P(B_1|G_3)$, $P(G_1|G_3)$, $P(B_1|B_3)$ and $P(G_1|B_3)$.

c) (1 mark) Are B_1 and B_3 independent? Justify your answer.

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2. (3 marks total) To gain practical knowledge of ECE 302, you find a quiet stairway in the new Bahen Centre, position yourself just below the first step (height equal to 0), blind-fold yourself, and then randomly jump up and down while counting jumps out loud. The probability that you jump up to the next step is p, while the probability that you jump onto the same step you're currently on is $1-p$ (amazingly, you don't jump down steps). You jump n times, take off your blind-fold, and note your height, which is a random variable X, measured in "steps". There's a catch: Unfortunately, the stairway has not yet been completed. There are only k steps ($k < n$), followed by a sheer drop back down to a height of 0. So, if you're on step $k-1$ and jump "up", instead of landing on step k, you fall down to ground level (height equal to 0), terminate the experiment early, and note a height of 0 in your lab book.

a) (1 mark) Specify the sample space S_X of the random variable X and derive the probability mass function $P(X = j)$ for $j \in S_X$.

b) (1 mark) For $p = 1/3$, $n = 5$, $k = 3$, write down the probabilities $P(X = j)$ for $j \in S_X$ as simplified fractions, and draw the cumulative distribution function.

c) (1 mark) Is X a discrete, a continuous, or a mixed random variable?

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3. (4 marks total) The "Beta random variable" X has a sample space $S_X = [0, 1]$ and is useful for modeling numbers that are in a finite interval. For a specific choice of parameters, the pdf for X is $f_X(x) = cx(1-x)^3, \quad 0 \leq x \leq 1$.

a) (1 mark) Determine the numerical value of the constant c.

b) (1 mark) What value of x maximizes the density? What is the maximum density?

c) (1 mark) Sketch the pdf of X.

d) (1 mark) Derive the cdf $F_X(x)$ *and* sketch the cdf.