

ECE302F Lecture Group 04 Quiz 2

Rules:

- No books or aid sheets of any sort allowed;
 - Non-programmable electronic calculators CAN be used;
 - Total duration of quiz: 30 minutes;
 - Answer all questions.
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1. Consider the function

$$f_X(x) = \begin{cases} ax^2 + bx & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where a and b are constants.

- (a) Sketch $f_X(x)$ for the four cases $\{a < 0, b < 0\}$, $\{a < 0, b > 0\}$, $\{a > 0, b > 0\}$, and $\{a > 0, b < 0\}$. Hence, give the condition(s) a and b must satisfy so that $f_X(x) \geq 0$ in the range $0 \leq x \leq 1$. (3 marks)
- (b) What condition apart from those in part (a) must a and b satisfy in order for $f_X(x)$ to be a valid pdf? (2 marks)
2. A children's toy consists of a 4×4 grid of lights. Upon punching a button, a random number of squares will light up. Each square has a probability of 0.1 of turning on when the button is punched. The squares light up independently of one another. Give the name of the distribution of the random variable

$X =$ "Number of squares turned on after hitting the button".

Hence find $P(X = 3)$. (2 marks)

The toy manufacturer, in a recent re-vamp of the product, increases the number of squares to 64 in an 8×8 grid. Each square now lights up with a probability of 0.04. Compute the probability that 5 squares light up using the exact method above, and an approximate method. (3 marks)