

ECE302F Lecture Groups 101 and 102 – Quiz 1

Rules:

- No books or aid sheets of any sort allowed;
 - Non-programmable electronic calculators CAN be used;
 - Total duration of quiz: 30 minutes;
 - Answer all questions. Each question carries equal marks.
-

1. Use Venn diagrams to prove De Morgan's laws $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$, where the superscript c denotes the complement of a set. You should use one Venn diagram to represent the left-hand side of each equation, and one more to represent the right-hand side.
2. If A is independent of B , show that A^c is independent of B^c .
3. Prove that a set with N elements has 2^N subsets, inclusive of the the null set and the set itself. The binomial expansion $(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$ may be useful.
4. A frequently held lottery sells 100 tickets at \$1 per ticket every time it is held. One of the tickets must be a winner. A player has \$50 to spend. To maximize the probability of winning at least one lottery, should he buy 50 tickets in one lottery or one ticket in 50 lotteries? Explain your answer.
5. Consider the following relay network. The network functions if and only if there is a closed path of relays from left to right. Assume that the relays fail independently and that the probability of failure for each relay is as shown. What is the probability that the relay network operates?

