

ECE221S – Quiz 5B – Wednesday, April 2, 2003

Family Name, Given Name(s)	Student Number
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Please insert your name in full and your student number.

An infinite cylindrical shell with inner radius a , and outer radius b , consisting of a material with $\mu = \mu_r \mu_0$ for $a \leq \rho \leq b$, is oriented such that the z -axis of a cylindrical coordinate system coincides with its axis. The medium anywhere else is free space. Within the cylindrical shell the magnetic field intensity is $\underline{H}_1 = 3\hat{\rho} + 5\hat{z}$ [mA/m]. On the outside of the shell ($\rho > b$) the magnetic field intensity is $\underline{H}_2 = 15\hat{\rho} + 20\hat{z}$ [mA/m].

- (a) Find the relative permeability of the medium in the cylindrical shell. **(1 mark)**
- (b) Determine the surface current density \underline{K} on the outside of the cylindrical shell. **(2 marks)**
- (c) Find the magnetic field intensity for the space surrounded by the cylindrical shell ($0 < \rho < a$) under the assumption that the surface current density on the inside of the shell is $-\underline{K}$, using your result from part (b) for \underline{K} . **(2 marks)**

Useful equations: $B_{1n} = B_{2n}$ $(\underline{H}_1 - \underline{H}_2) \times \hat{n} = \underline{K}$ (\hat{n} points from material 1 to 2).

(a) $B_{1n} = B_{2n} \rightarrow \mu_0 \mu_r H_{1z} = \mu_0 H_{2z} \rightarrow \mu_r = \frac{H_{2z}}{H_{1z}} = \frac{15}{3} = 5$ $\mu_r = 5$

(b) $\hat{n} = \hat{z}$
 $(3\hat{\rho} + 5\hat{z} - 15\hat{\rho} - 20\hat{z}) \times \hat{z} = -15\hat{\phi} = \underline{K}$ $\underline{K} = -15\hat{\phi}$ [mA/m]

(c) \underline{H}_3 : magnetic field intensity for ($0 < \rho < a$)

$$(\underline{H}_3 - \underline{H}_1) \times \hat{z} = -\underline{K} \rightarrow (H_{3z} - 5)\hat{\phi} = 15\hat{\phi}$$

$$\leadsto H_{3z} = 20 \text{ [mA/m]}$$

$$\mu_0 H_{3n} = B_{3n} = B_{3s} = B_{1n} = \mu_0 \mu_r H_{1s} = \mu_0 \cdot 5 \cdot 3 \rightarrow H_{3s} = 15 \text{ [mA/m]}$$

$$\underline{H}_3 = (15\hat{s} + 20\hat{z}) \text{ [mA/m]} = \underline{H}_2$$