ECE221S – Quiz 5A – Monday, March 31, 2003

Family Name, Given Name(s)	Student Number

Please insert your name in full and your student number.

An infinite cylinder of radius a consisting of a material with $\mu=3\mu_0$ is oriented such that the z-axis of a cylindrical coordinate system coincides with its axis. The cylinder is surrounded by free space. The magnetic flux density inside the cylinder is $\underline{B} = 15\hat{\underline{\rho}} - 6\hat{\underline{\phi}}$ [mT]. The cylinder surface carries a surface current density of $\underline{K} = 3\hat{\underline{z}}$ [mA].

(a) Find the magnetic field intensity $\underline{H}$ inside the cylinder. **(1 mark)**
(b) Calculate the magnetization $\underline{M}$ inside the cylinder. **(1 mark)**
(c) Find the fields $\underline{H}$ and $\underline{B}$ outside of the cylinder. **(3 marks)**

Useful equations: $B_{1n}=B_{2n}$ $\quad (\underline{H}_1 - \underline{H}_2)\times\hat{\underline{n}} = \underline{K}$ ($\hat{\underline{n}}$ points from material 1 to 2).

(a) $\underline{H} = \dfrac{\underline{B}}{3\mu_0} = \dfrac{5}{\mu_0}\hat{\underline{s}} - \dfrac{2}{\mu_0}\hat{\underline{\varphi}} = (3.98\,\hat{\underline{s}} - 1.59\,\hat{\underline{\varphi}})$ [kA/m]

(b) $\underline{M} = \chi_m \underline{H} \qquad \chi_m = \mu_r - 1 = 2$

$\underline{M} = \dfrac{10}{\mu_0}\hat{\underline{s}} - \dfrac{4}{\mu_0}\hat{\underline{\varphi}} = (15.9\,\hat{\underline{s}} - 6.37\,\hat{\underline{\varphi}})\ 10^3$ [A/m]

(c) $B_{1n} = B_{2n} \rightarrow \underline{B}_{2n} = 15\,\hat{\underline{s}}$ [mT] $\qquad \hat{\underline{n}} = \hat{\underline{s}}$

$(\underline{H} - \underline{H}_2)\times\hat{\underline{s}} = \left(-\dfrac{2}{\mu_0}\hat{\underline{\varphi}} - H_{2\varphi}\right)\times\hat{\underline{s}} = \left(\dfrac{2}{\mu_0} + H_{2\varphi}\right)\hat{\underline{z}} = 3\,\hat{\underline{z}}$

$\rightarrow H_{2\varphi} = \left(3 - \dfrac{2}{\mu_0}\right)$ [mA/m] ≈ 1.59 [kA/m]

$H_{2s} = \dfrac{B_{2s}}{\mu_0} = \dfrac{15}{\mu_0} \approx 11.9$ [kA/m]

$B_{2\varphi} = \mu_0 H_{2\varphi} = 3\mu_0 - 2 = -1.999996$ [mT]

$\underline{H}_2 = (1.59\,\hat{\underline{\varphi}} + 11.9\,\hat{\underline{s}})$ [kA/m]

$\underline{B}_2 = (-1.999996\,\hat{\underline{\varphi}} + 15\,\hat{\underline{s}})$ [mT]