

ECE221S – Quiz 4B – Wednesday, March 19, 2003

Family Name Given Name(s)	Student Number
---------------------------	----------------

Please Insert Your Name in Full and Student Number

1. Using Ampere's law $\left(\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{encl} \right)$ find the magnetic field due to an infinitely long wire (placed along the z-axis) carrying current I . **(1.5 marks)**
2. Two infinite wires are separated by 10cm. They carry equal but opposite currents of 200A. Find the flux per unit length passing through the plane of the conductors. ($\mu_0 = 4\pi \times 10^{-7}$) **(3 marks)**
3. What would the flux be if the two currents were in the same direction? **(0.5 marks)**

Solution: 1): This question was done in class. From our usual application we know that the magnetic field \mathbf{H} is in the \mathbf{f} direction. As done in class choosing a circular loop of radius r , centered at the wire,

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = H_f 2\pi r \cdot I_{encl} = I \Rightarrow \mathbf{H} = \frac{I}{2\pi r} \mathbf{a}_f$$

2. The magnetic flux density $\mathbf{B} = \mu_0 \mathbf{H}$. The flux through the plane is therefore $\mathbf{y} = \int_S \mathbf{B} \cdot d\mathbf{s}$. In this case, $d\mathbf{s} = dr dz \mathbf{a}_f$. Now, since the currents are equal and in opposite directions, by symmetry, the total flux is twice the flux due to a single wire. The flux due to one wire is given by

$$\int_S \mathbf{B} \cdot d\mathbf{s} = \int_{z_1}^{z_2} \int_{r_1}^{r_2} \frac{\mu_0 I}{2\pi r} dr dz = (z_2 - z_1) \frac{\mu_0 I}{2\pi} \ln(r) \Big|_{r_1}^{r_2} = (z_2 - z_1) \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_2}{r_1}\right) = \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

At this point we're stuck. Clearly $\rho_2 = 10\text{cm}$, but $\rho_1 = 0$, leading to an infinite flux. What I should have done was given the wires a radius which would have made ρ_1 non-zero. Arbitrarily choosing $\rho_1 = 1\text{cm}$, the flux per unit length is given by

$$\mathbf{y} = 2 \times \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_2}{r_1}\right) = 2 \times \frac{4\pi \times 10^{-7} \times 200}{2\pi} \ln\left(\frac{10}{1}\right) = (8 \times 10^{-5}) \ln(10) = 1.84 \times 10^{-4} \text{ Wb/m}$$

3. Since the currents are in the same direction, the fields now are in opposition. The flux due to each current source cancels out leading to **zero** flux between the wires.