

Dterm ECE221S – Quiz 4A –Monday, March 17, 2003

Family Name Given Name(s)	Student Number
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Please Insert Your Name in Full and Student Number

1. Using Ampere's law $\left(\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{encl} \right)$ find the magnetic field due to an infinitely long wire (placed along the z-axis) carrying current I .
(2 marks)
2. If $I = 100\text{A}$, calculate the total flux passing through a plane bounded by $r = 1\text{cm}$, $r = 10\text{cm}$, $z = 5\text{cm}$ and $z = 50\text{cm}$. $\mu_0 = 4\pi \times 10^{-7}$ (3 marks)

Solution: 1): This question was done in class. From our usual application we know that the magnetic field \mathbf{H} is in the \mathbf{f} direction. As done in class choosing a circular loop of radius r , centered at the wire,

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = H_f 2\pi r \cdot I_{encl} = I \Rightarrow \mathbf{H} = \frac{I}{2\pi r} \mathbf{a}_r$$

2. The magnetic flux density $\mathbf{B} = \mu_0 \mathbf{H}$. The flux through the plane is therefore

$$\psi = \int_S \mathbf{B} \cdot d\mathbf{s} \text{ . In this case, } d\mathbf{s} = dr dz \mathbf{a}_f$$

$$\int_S \mathbf{B} \cdot d\mathbf{s} = \int_{z_1}^{z_2} \int_{r_1}^{r_2} \frac{\mu_0 I}{2\pi r} dr dz = (z_2 - z_1) \frac{\mu_0 I}{2\pi} \ln(r) \Big|_{r_1}^{r_2} = (z_2 - z_1) \frac{\mu_0 I}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

$$= (50 - 5) \times 10^{-2} \times \frac{4\pi \times 10^{-7} \times 100}{2\pi} \ln(10) = 9 \times 10^{-6} \ln(10) = 2.07 \times 10^{-5} \text{ Wb}$$