ECE221S – Quiz 3A – Monday, March 3, 2003

Family Name, Given Name(s)	Student Number

Please insert your name in full and your student number.

Consider a section of a spherical shell with inner radius a and outer radius b which ranges from $\theta=0°$ to $\theta=90°$ and from $\varphi=0°$ to $\varphi=90°$. The potential at $r=a$ is V_o and the potential at $r=b$ is 0. This region is filled with a material with conductivity σ and permittivity ε. a and b are 10cm and 20cm respectively.

(a) Find the potential at any point in the material between $r=a$ and $r=b$. Assume that the potential depends on the radial variable only.
(b) Assume the material to be copper ($\sigma=5.8\ 10^7$ S/m) and calculate the resistance between the curved surfaces at $r=a$ and $r=b$.
(c) Assume two curved metal surfaces bordering the material at $r=a$ and $r=b$ and the material to be polystyrene ($\varepsilon_r=2.55$), forming a capacitor.

Useful equations:

1. $\nabla V=\frac{\partial V}{\partial r}\underline{\hat{r}}+\frac{1}{r}\frac{\partial V}{\partial \theta}\underline{\hat{\theta}}+\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\underline{\hat{\phi}}$; $\quad \nabla^2 V=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$

2. For a *fixed* resistance/capacitance structure, $RC=\varepsilon/\sigma$. (Be careful how you use this!)

(a) $\nabla^2 V = 0 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) \underset{r\neq 0}{\curvearrowright} \frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)=0$

$\underset{\text{integrate}}{\curvearrowright} \frac{\partial V}{\partial r}=\frac{C_1}{r^2} \underset{\text{integrate}}{\curvearrowright} \boxed{V(r)=-\frac{C_1}{r}+C_2}$

Boundary conditions:

$V(r=a)=V_o=-\frac{C_1}{a}+C_2$
$V(r=b)=0=-C_1/b+C_2$

$\left.\begin{array}{l} C_2 = C_1/b \\ V_o=-C_1/a+C_1/b \end{array}\right\}\quad C_1=-\frac{V_o}{1/a-1/b}, \quad C_2=-\frac{V_o/b}{1/a-1/b}$

$\curvearrowright \boxed{V(r)=\frac{V_o}{1/a-1/b}\left(\frac{1}{r}-\frac{1}{b}\right)}$ $\quad a\le r\le b$, $0\le\theta\le 90°$, $0\le\varphi\le 90°$

(b) $\underline{E}(r)=-\nabla V=-\underline{\hat{r}}\frac{\partial V}{\partial r}=\frac{V_o}{1/a-1/b}\frac{1}{r^2}\underline{\hat{r}}$

$R=\frac{V_o}{I}$

$I=\sigma\int \underline{E}\cdot d\underline{s} = \sigma\int_{\theta=0}^{\pi/2}\int_{\varphi=0}^{\pi/2}\frac{V_o}{1/a-1/b}\frac{1}{r^2}\underline{\hat{r}}\cdot r^2\sin\theta\, d\theta\, d\varphi\, \underline{\hat{r}}$

$$I = \sigma \frac{V_0}{1/a - 1/b} \cdot \frac{\pi}{2} \left\{ -\cos\left(\frac{\pi}{2}\right) + \cos(0) \right\} = \frac{\sigma V_0 \pi}{2(1/a - 1/b)}$$

$$\boxed{R = \frac{2(1/a - 1/b)}{\sigma \pi}}$$

(c) $$C = \frac{\varepsilon}{\sigma} \frac{1}{R} = \frac{\varepsilon \pi}{2(1/a - 1/b)}$$