ECE221S – Quiz 2B –Wednesday, February 12, 2003

Family Name Given Name(s)	Student Number

Please Insert Your Name in Full and Student Number

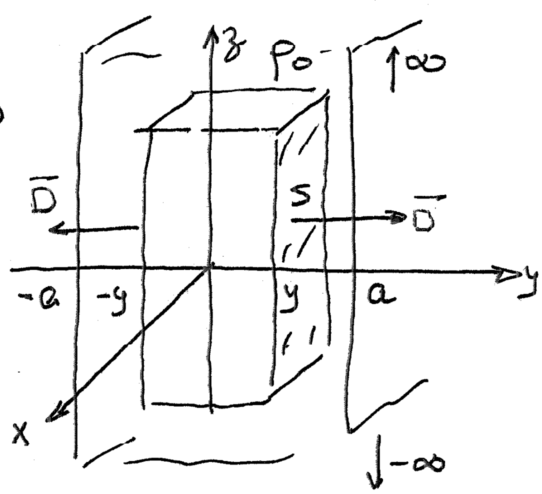
Consider an infinite slab of charge having a constant volume charge density ρ_0 [C/m^3]. The slab has thickness 2a and occupies the volume $-a \le y \le +a$; $-\infty < x,z < \infty$. The permittivity is that of free space everywhere.

a. Using suitably chosen Gaussian surfaces, develop expressions for the electric field strength **E** (magnitude and direction) as a function of y.

b. Letting $\rho_0 = 10$ C/m^3 and a = 1 μm [= 10^{-6} m], <u>accurately</u> plot the magnitude of the electric field for y in the range $-2a < y < +2a$.

c. For the values given in (b), determine the potential of a point A, having co-ordinates (2a, 2a, a), with respect to a point B, having the co-ordinates (5a, 0, 3a).

a)



Choose Gaussian surface shown within charged volume.

By nature of the charge distribution,

$\bar{E} = E_y(y)\,\hat{a}_y$

$\bar{D} = D_y(y)\,\hat{a}_y$

$$\oint_S \bar{D}\cdot d\bar{s} = 2S\,D_y(y)$$

Note: only surfaces at $\pm y$ contribute. For all other surfaces $\bar{D}\cdot d\bar{s} = 0$

$$Q_{encl} = \int \rho_v\, dv = \rho_0 \underbrace{(2yS)}_{\text{volume}}$$

$\therefore$ $\boxed{D_y = \rho_0 y}$ $\quad -a \le y \le a$ $\qquad \bar{E} = \dfrac{\rho_0 y}{\epsilon_0}\hat{a}_y \quad -a \le y \le a$

For $|y| \geq a$, Gauss's Law yields

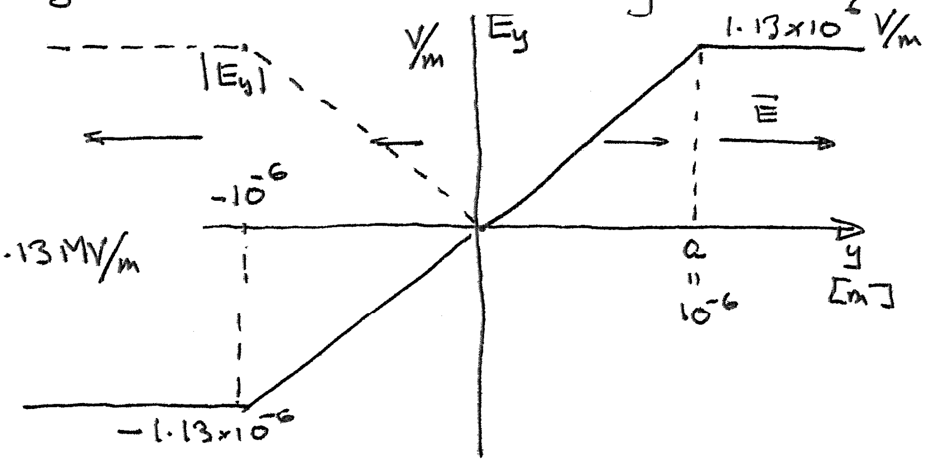
$$2SD_y(y) = \rho_0 (2a) S \implies \bar{D} = \rho_0 a \; \hat{a}_y \quad C/m^2 \quad y \geq a$$
$$= \rho_0 a (-\hat{a}_y) \quad C/m^2 \quad y \leq a$$

$$\bar{E} = \underline{\underline{\frac{\rho_0 a}{\epsilon_0} \hat{a}_y}} \quad y \geq a \quad \text{and} \quad \bar{E} = \underline{\underline{-\frac{\rho_0 a}{\epsilon_0} (\hat{a}_y)}} \quad y \leq -a$$

In the external region, $\bar{E}$ is constant in magnitude

b) $\rho_0 = 10 \; C/m^3$
$a = 10^{-6}$ m

$$\frac{\rho_0 a}{\epsilon_0} = \frac{10 \times 10^{-6}}{8.85 \times 10^{-12}} = 1.13 \; MV/m$$



The above diagram indicated the direction and magnitude (-ve being in the $-\hat{a}_y$ direction

Absolute value is shown as a dotted line for $y \leq 0$

c) $A[2a, 2a, a]$ Note: planes of constant y are
$B[5a, 0, 3a]$ equipotentials.

We want potential of A w.r.t B. Therefore

$$V_A - V_B = -\int_B^A \bar{E} \cdot d\bar{l} = -\int_0^{2a} E_y \, dy = -\int_0^a \frac{\rho_0 y}{\epsilon} dy - \int_a^{2a} \frac{\rho_0 a}{\epsilon_0} dy$$

$$V_A - V_B = -\frac{3}{2}\frac{\rho_0 a^2}{\epsilon_0} = -\frac{3}{2} \times \frac{10 \times 10^{-12}}{8.85 \times 10^{-12}} = -\underline{\underline{1.695}} V$$