ECE221S – Quiz 2A –Monday, February 10, 2003

Family Name Given Name(s)	Student Number

Please Insert Your Name in Full and Student Number

Consider a spherical region of radius R centered at the origin of a co-ordinate system. The total charge within the region is Q.

Within the spherical region, the volume charge density ρ_v is proportional to the radial position r; i.e. $\rho_v = kr$. It can be shown that the constant k is: $k = Q/(\pi R^4)$.

a. Consider a point A on the surface of the spherical region. Determine the potential of that point relative to:

   i. A point at infinity.

   ii. A point at the center of the spherical region.

b. Let the total charge within the spherical region equal be $Q = -1.6 \times 10^{-19}$ C, which is the charge of an electron. Let the radius of the region be $R = 10^{-10}$ m. For these values, determine the potential of a point on the surface of the spherical region with respect to the central point of the region.

BACKGROUND (NOT REQUIRED FOR A COMPLETE SOLⁿ).

$\rho_v = kr \quad [C/m^3]$ TOTAL CHARGE $Q = \int_V dQ$

In a shell of radius r, thickness dr:

$dQ = \rho_v dv = 4\pi k r^3 dr$

$$Q = \int_0^R 4\pi k r'^3 dr' = \pi k R^4 \implies \boxed{k = Q/(\pi R^4)}$$

a) To find the potential V, it is first necessary to find $\bar{E}$ everywhere.

For points $r \geq R$, by Gauss's Law $\oint_S \bar{D} \cdot d\bar{s} = Q_{encl} = Q$

Choosing a spherical surface centered at origin for Gaussian surface:

$$\oint_S \bar{D}\cdot d\bar{s} = \int_S D\,ds = 4\pi r^2 D = Q$$

$\bar{D} = D(r)\,\hat{a}_r \qquad \bar{E} = \bar{D}/\epsilon_0 \Rightarrow \boxed{\bar{E} = \dfrac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r} \quad r \geq R$

$r < R \qquad Q_{encl} = \displaystyle\int_0^r 4\pi k r'^3\,dr' = \pi k r^4 = Q\left(\frac{r}{R}\right)^4$

By Gauss's Law: $\quad 4\pi r^2 D(r) = Q\left(\dfrac{r}{R}\right)^4$

$$\boxed{\bar{E} = \frac{Q}{4\pi\epsilon_0}\,\frac{r^2}{R^4}\,\hat{a}_r}$$

$\boxed{a\text{-}i}$ Potential of point A on surface with respect to point B at infinity:

$$V_A - V_B = -\int_B^A \bar{E}\cdot d\bar{l} = -\int_\infty^R \frac{Q}{4\pi\epsilon_0 r^2}\,dr \Rightarrow \boxed{V_A - V_\infty = \frac{Q}{4\pi\epsilon_0 R}}$$

$\boxed{a\text{-}ii}$ Potential of point A on surface of charge distribution w.r.t point at $r = 0$

$$V_A - V_B : V_A - V(0) = -\int_0^R \frac{Q}{4\pi\epsilon_0 R^4}\,r^2\,dr \Rightarrow \boxed{-\frac{Q}{4\pi\epsilon_0}\left(\frac{1}{3R}\right) = V_A - V_0}$$

b) $Q = -1.6\times10^{-19}\,C$
$R = 10^{-10}\,m$

$$V_A - V_0 = +\frac{1.6\times10^{-19}}{4\pi\times8.85\times10^{-12}}\times\frac{1}{3\times10^{-10}}$$

$$\boxed{V_A - V_0 = 4.796V}$$

i.e. The surface is 4.8 V above center.