ECE221S QUIZ No.1A **Monday, January 27, 2003**

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(1) Consider a *cylindrical* coordinate system (ρ,ϕ,z) in which point P is located at (50m, 30°, 10m) and point P' is located at (42m, 45°, 27m). The position vectors of P and P' in the *cylindrical* coordinate system are $\mathbf{r} = 50\hat{\mathbf{a}}_\rho + 10\hat{\mathbf{a}}_z$ [m] and $\mathbf{r}' = 42\hat{\mathbf{a}}_\rho + 27\hat{\mathbf{a}}_z$ [m], respectively. Find

(a) the position vectors **r** and **r'** in the *Cartesian* coordinate system.
(b) the distance vector $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ in the *Cartesian* coordinate system.
(c) the distance vector $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ at point P in the *cylindrical* coordinate system
(d) the distance vector $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ at point P' in the *cylindrical* coordinate system

Hint: Make use of the Table given below.

(2) Consider a *cylindrical* coordinate system (ρ,ϕ,z) in which point charges $Q_1 = 1$ μC and $Q_2 = 2$ μC are located at points $P_1' = (42\text{m}, 45°, 27\text{m})$ and $P_2' = (-10\text{m}, -90°, 7\text{m})$, respectively. The medium is free space with permittivity $\varepsilon_o = 8.854\times10^{-12}$ F/m. Use vector calculus to find the electric field intensity vector **E** at the point P = (50m, 30°, 10m) from the following expression

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{2}\frac{Q_i(\mathbf{r}-\mathbf{r}_i')}{|\mathbf{r}-\mathbf{r}_i'|^3}$$

Carry out the superposition implied by the summation in the *cylindrical* coordinate system.

(1) Solution:[1]

(a) In the *Cartesian* coordinate system:

$$P = [50\cos(30°), 50\sin(30°), 10] = (43.30, 25.00, 10.00)$$
$$\mathbf{r} = 43.30\hat{\mathbf{a}}_x + 25.00\hat{\mathbf{a}}_y + 10.00\hat{\mathbf{a}}_z \text{ [m]}$$

$$P' = [42\cos(45°), 42\sin(45°), 27] = (29.70, 29.70, 27.00)$$
$$\mathbf{r}' = 29.70\hat{\mathbf{a}}_x + 29.70\hat{\mathbf{a}}_y + 27.00\hat{\mathbf{a}}_z \text{ [m]}$$

(b) In the *Cartesian* coordinate system:

$$\mathbf{R} = \mathbf{r} - \mathbf{r}' = 43.30\hat{\mathbf{a}}_x + 25.00\hat{\mathbf{a}}_y + 10.00\hat{\mathbf{a}}_z - 29.70\hat{\mathbf{a}}_x - 29.70\hat{\mathbf{a}}_y - 27.00\hat{\mathbf{a}}_z$$
$$= 13.60\hat{\mathbf{a}}_x - 4.70\hat{\mathbf{a}}_y - 17.00\hat{\mathbf{a}}_z \text{ [m]}$$

(c) In the *cylindrical* coordinate system:

At P: $\mathbf{R} = \mathbf{r} - \mathbf{r}' = [+13.60\cos(30°) - 4.70\sin(30°)]\hat{\mathbf{a}}_\rho$

[1] See textbook for relationship between vector components in Cartesian, cylindrical and spherical coordinate systems.

$$[-13.60\sin(30°) - 4.70\cos(30°)]\hat{\mathbf{a}}_\phi$$
$$[-17.00]\hat{\mathbf{a}}_z$$
$$= +9.43\hat{\mathbf{a}}_\rho - 10.87\hat{\mathbf{a}}_\phi - 17.00\hat{\mathbf{a}}_z \text{ [m]}$$

(d) In the *cylindrical* coordinate system:

At P': $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ $= [+13.60\cos(45°) - 4.70\sin(45°)]\hat{\mathbf{a}}_\rho$

$$[-13.60\sin(45°) - 4.70\cos(45°)]\hat{\mathbf{a}}_\phi$$
$$[-17.00]\hat{\mathbf{a}}_z$$
$$= +6.29\hat{\mathbf{a}}_\rho - 12.94\hat{\mathbf{a}}_\phi - 17.00\hat{\mathbf{a}}_z \text{ [m]}$$

(2) Solution:

Field point and position vector in *Cartesian* coordinates:

P = [50cos(30°), 50sin(30°), 10] = (43.30, 25.00, 10.00)

$\mathbf{r}$ $= 43.30\hat{\mathbf{a}}_x + 25.00\hat{\mathbf{a}}_y + 10.00\hat{\mathbf{a}}_z$ [m]

Source point and position vector No.1 in *Cartesian* coordinates:

P_1' = [42cos(45°), 42sin(45°), 27] = (29.70, 29.70, 27.00)

$\mathbf{r}_1$' $= 29.70\hat{\mathbf{a}}_x + 29.70\hat{\mathbf{a}}_y + 27.00\hat{\mathbf{a}}_z$ [m]

Distance vector No.1 in *Cartesian* coordinates:

$$\mathbf{R}_1 = \mathbf{r} - \mathbf{r}_1' = 43.30\hat{\mathbf{a}}_x + 25.00\hat{\mathbf{a}}_y + 10.00\hat{\mathbf{a}}_z - 29.70\hat{\mathbf{a}}_x - 29.70\hat{\mathbf{a}}_y - 27.00\hat{\mathbf{a}}_z$$
$$= 13.60\hat{\mathbf{a}}_x - 4.70\hat{\mathbf{a}}_y - 17.00\hat{\mathbf{a}}_z \text{ [m]}$$

Distance vector No.1 in *cylindrical* coordinates:

At P: $\mathbf{R}_1 = \mathbf{r} - \mathbf{r}_1' =$ $[+13.60\cos(30°) - 4.70\sin(30°)]\hat{\mathbf{a}}_\rho$

$$[-13.60\sin(30°) - 4.70\cos(30°)]\hat{\mathbf{a}}_\phi$$
$$[-17.00]\hat{\mathbf{a}}_z$$
$$= +9.43\hat{\mathbf{a}}_\rho - 10.87\hat{\mathbf{a}}_\phi - 17.00\hat{\mathbf{a}}_z \text{ [m]}$$

Source point and position vector No.2 in *Cartesian* coordinates:

P_2' = [–10cos(–90°), –10sin(–90°), 7] = (0.00, 10.00, 7.00)

$\mathbf{r}_2$' $= 0.00\hat{\mathbf{a}}_x + 10.00\hat{\mathbf{a}}_y + 7.00\hat{\mathbf{a}}_z$ [m]

Distance vector No.2 in *Cartesian* coordinates:

$$\mathbf{R}_2 = \mathbf{r} - \mathbf{r}_2' = 43.30\hat{\mathbf{a}}_x + 25.00\hat{\mathbf{a}}_y + 10.00\hat{\mathbf{a}}_z - 0.00\hat{\mathbf{a}}_x - 10.00\hat{\mathbf{a}}_y - 7.00\hat{\mathbf{a}}_z$$

$$= 43.30\hat{\mathbf{a}}_x + 15.00\hat{\mathbf{a}}_y + 3.00\hat{\mathbf{a}}_z \text{ [m]}$$

Distance vector No.2 in *cylindrical* coordinates

At P: $\mathbf{R_2} = \mathbf{r} - \mathbf{r_2}' =$
$$[+43.30\cos(30°) + 15.00\sin(30°)]\hat{\mathbf{a}}_\rho$$
$$[-43.30\sin(30°) + 15.00\cos(30°)]\hat{\mathbf{a}}_\phi$$
$$[+3.00]\hat{\mathbf{a}}_z$$
$$= +37.50\hat{\mathbf{a}}_\rho + 7.50\hat{\mathbf{a}}_\phi + 3.00\hat{\mathbf{a}}_z \text{ [m]}$$

Electric field intensity vector in *cylindrical* coordinates:

$$\mathbf{E} = \frac{\mathbf{Q}_1}{4\pi\varepsilon_\mathbf{o}} \frac{(\mathbf{r} - \mathbf{r}_1^{'})}{|\mathbf{r} - \mathbf{r}_1^{'}|^3} + \frac{\mathbf{Q}_2}{4\pi\varepsilon_\mathbf{o}} \frac{(\mathbf{r} - \mathbf{r}_2^{'})}{|\mathbf{r} - \mathbf{r}_2^{'}|^3}$$

$$= \frac{(1\times10^{-6})[\mathbf{C}]}{4\pi(8.854\times10^{-12}[\mathbf{F/m}])} \frac{(+9.43\hat{\mathbf{a}}_\rho - 10.87\hat{\mathbf{a}}_\phi - 17.00\hat{\mathbf{a}}_z)[\mathbf{m}]}{(22.27)^3[\mathbf{m}^3]}$$

$$+ \frac{(2\times10^{-6})[\mathbf{C}]}{4\pi(8.854\times10^{-12}[\mathbf{F/m}])} \frac{(+37.50\hat{\mathbf{a}}_\rho + 7.50\hat{\mathbf{a}}_\phi + 3.00\hat{\mathbf{a}}_z)[\mathbf{m}]}{(38.36)^3[\mathbf{m}^3]}$$

$$= \frac{[\mathbf{N}]}{4\pi(8.854\times10^{-6})[\mathbf{C}]} \left[\frac{(9.43\hat{\mathbf{a}}_\rho - 10.87\hat{\mathbf{a}}_\phi - 17.00\hat{\mathbf{a}}_z)}{11{,}044.87} + \frac{2(37.50\hat{\mathbf{a}}_\rho + 7.50\hat{\mathbf{a}}_\phi + 3.00\hat{\mathbf{a}}_z)}{56{,}446.34} \right]$$

$$= \frac{853.79\hat{\mathbf{a}}_\rho - 984.17\hat{\mathbf{a}}_y - 1{,}539.18\hat{\mathbf{a}}_z + 1{,}328.70\hat{\mathbf{a}}_\rho + 265.74\hat{\mathbf{a}}_\phi + 106.30\hat{\mathbf{a}}_z}{35.416\pi} \quad \textbf{[N/C]}$$

$$= \frac{2{,}182.49\hat{\mathbf{a}}_\rho - 718.43\hat{\mathbf{a}}_\phi - 1{,}432.88\hat{\mathbf{a}}_z}{35.416\pi} \quad \textbf{[N/C]}$$

$$= 19.62\hat{\mathbf{a}}_\rho - 6.46\hat{\mathbf{a}}_\phi - 12.88\hat{\mathbf{a}}_z \quad \textbf{[N/C]}$$