

Last Name, First Name

Student No.

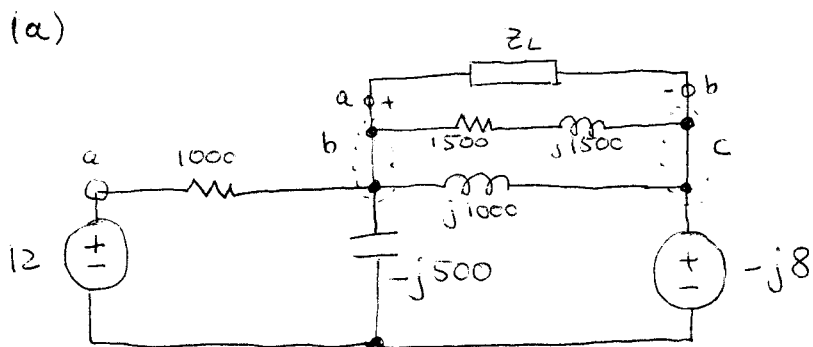
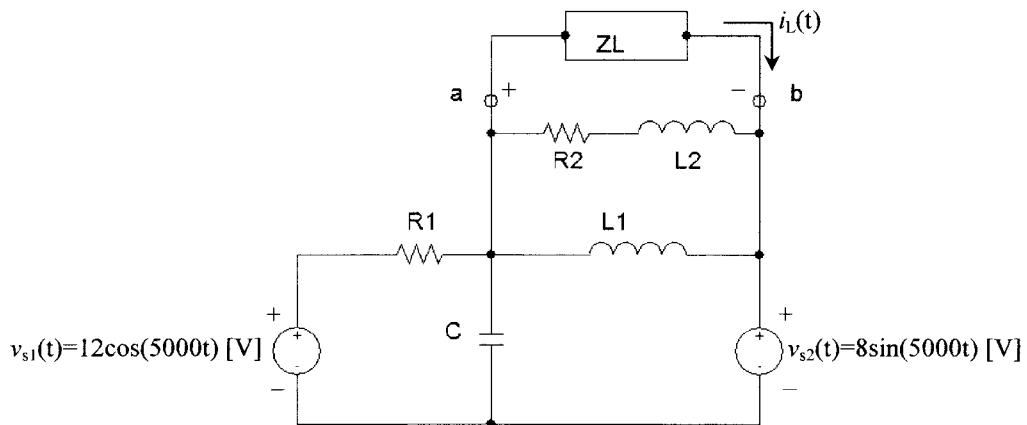
Check your tutorial section(**REQUIRED!**):

TUT0101	TUT0102	TUT0103	TUT0104	TUT0105	TUT0106	TUT0107	TUT0108
Thu. 9 am	Thu. 9 am	Thu. 9 am	Thu. 9 am	Mon. 10am	Mon. 10am	Mon. 3 pm	Mon. 3 pm
BA2165	BA2155	BA2145	BA2195	BA2145	BA2165	BA2145	HA410
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University of Toronto
Department of Electrical & Computer Engineering
ECE212F – Circuit Theory
Quiz 2 – November 6, 2002, 6:30-7:30 PM

Instructions: Non-programmable calculators allowed. No other aids. Answer in the space provided on these sheets. The back sides of these sheets can be used as well. For full marks (25) you must show methods, state UNITS and compute numerical answers when requested. *Please write in pen, NOT IN PENCIL! Quizzes written in pencil can not be considered for mark revision.*

1. **Sinusoidal Steady State.** In the circuit below $R_1=1k\Omega$, $R_2=1.5k\Omega$, $L_1=200mH$, $L_2=300mH$, and $C=0.4\mu F$.
 - (a) Transform the circuit into phasor domain. (2 marks)
 - (b) Use the node-voltage or mesh-current method to find the phasor voltage between terminals a and b for $Z_L=\infty$ (open circuit). (6 marks)
 - (c) Use your result from part (b) to find the Thevenin equivalent circuit seen by the load, between terminals a and b. (2 marks)
 - (d) For a load impedance consisting of a 500Ω resistor in series with a $50mH$ inductor, calculate the load current $i_L(t)$. (2 marks)
 - (e) For a load impedance consisting of a 500Ω resistor in parallel with a $0.2\mu F$ capacitor, calculate the load current $i_L(t)$. (2 marks)



$$1.) (b) \quad V_a = 12, \quad V_c = -j8 \quad (\text{see part (a)})$$

$$\frac{V_a - V_b}{1000} + \frac{V_c - V_b}{1500 + j1500} + \frac{V_c - V_b}{j1000} = \frac{V_b}{-j500}$$

$$V_b \left(\frac{1}{1000} + \frac{1}{1500 + j1500} + \frac{1}{j1000} - \frac{1}{j500} \right) = \frac{V_a}{1000} + \frac{V_c}{1500 + j1500} + \frac{V_c}{j1000}$$

$$V_b \frac{1}{3000} (4 + 2j) = \frac{1}{3000} (4 - 8j)$$

$$V_b = -2j$$

$$V_{bc} = V_b - V_c = -2j + 8j = 6j$$

$$V_{oc} = 6j \text{ [V]}$$

$$(c) \quad V_T = V_{oc} = 6j \text{ [V]}$$

$$Z_T = \left(\frac{1}{1500 + j1500} + \frac{1}{j1000} - \frac{1}{j500} + \frac{1}{1000} \right)^{-1} = (600 - j300) \text{ [\Omega]}$$

$$(d) \quad Z_L = 500 + j250 \text{ [\Omega]}$$

$$I_L = \frac{V_T}{Z_T + Z_L} = (-0.2475 + j5.4433) \text{ [mA]}$$

$$= 5.4489 \angle 92.6^\circ$$

$$i_L(t) = 5.4489 \cos(5000t + 92.6^\circ) \text{ [mA]}$$

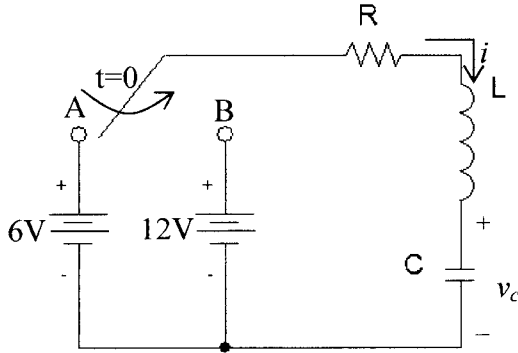
$$(e) \quad Z_L = 400 - j200$$

$$I_L = (-2.4 + j4.8) = 5.3666 \angle 116.6^\circ \text{ [mA]}$$

$$i_L(t) = 5.3666 \cos(5000t + 116.6^\circ) \text{ [mA]}$$

2. RLC circuit. In the circuit below $R=1k\Omega$, $L=0.5H$, and $C=10nF$. The switch has been in position A for a long time when it is switched to position B at $t=0$.

- (a) Find the initial and final values for the voltage $v_c(t)$ and the current $i(t)$. (2 marks)
 (b) Derive a differential equation in $v_c(t)$ for $t \geq 0$ and write down its characteristic equation. (3 marks)
 (c) Find the natural and forced responses, solve the equations from part (b) to find $v_c(t)$ and $i(t)$, and sketch $v_c(t)$ for $t \geq 0$. (5 marks)
 (d) Is the circuit over-, under-, or critically damped? Explain your answer. (1 mark)



(a) $v_c(0) = 6V$, $v_c(\infty) = 12V$, $i_L(0) = 0$, $i_L(\infty) = 0$

(b) $L \frac{di}{dt} + Ri + v_c = 12$ (KVL) $i = C \frac{dv_c}{dt}$

$LC \frac{d^2 v_c}{dt^2} + RC \frac{dv_c}{dt} + v_c = 12$

$$LC \left(\frac{d^2 v_c}{dt^2} + \frac{R}{L} \frac{dv_c}{dt} + \frac{1}{LC} v_c \right) = 12$$

$$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$$

$$s^2 + 2000s + 2 \cdot 10^8 = 0$$

(c) $s_{1/2} = -1000 \pm j14107$

$v_c(t) = e^{-1000t} \{ A \cos(14107t) + B \sin(14107t) \} + 12$ [V]

← natural response → forced response

$i(t) = C \frac{dv_c}{dt} = -1000 \cdot 10^{-8} e^{-1000t} \{ A \cos(14107t) + B \sin(14107t) \}$
 $+ 10^{-8} e^{-1000t} \cdot 14107 \{ -A \sin(14107t) + B \cos(14107t) \}$ [A]

$= e^{-1000t} \{ (10^{-5} A + 1.4107 \cdot 10^{-4} B) \cos(14107t) + (10^{-5} B - 1.4107 \cdot 10^{-4} A) \sin(14107t) \}$ [A]

$v_c(0) = A + 12 = 6 \rightarrow A = -6$

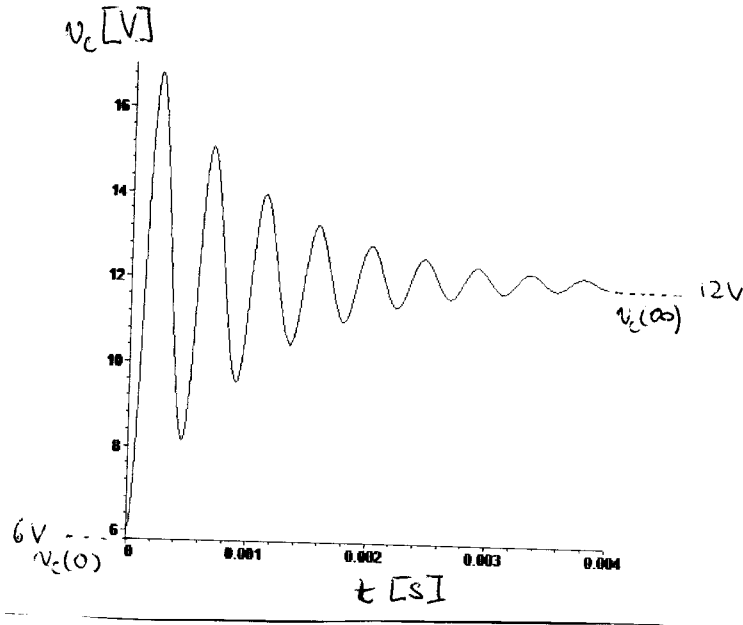
$i(0) = 0 \rightarrow 10^{-5} A + 1.4107 \cdot 10^{-4} B = 0 \rightarrow B = -0.425$

$$v_c(t) = e^{-1000t} \left\{ -6 \cos(14107t) - 0.425 \sin(14107t) \right\} + 12 \quad [\text{V}]$$

← natural
forced

$$i(t) = 0.85 e^{-1000t} \sin(14107t) \quad [\mu\text{A}]$$

← natural



(d) The circuit is underdamped, s_1, s_2 are complex conjugates.